Presented at the Illuminating Engineering Society Annual Technical Conference, Dallas, TX, August 24-28, 1980; and published in the Journal of the IES, July 1981, pp. 219-227

CALCULATING INTERIOR DAYLIGHT ILLUMINATION WITH A PROGRAMMABLE HAND CALCULATOR

H.J. Bryan and R.D. Clear

July 1981

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

LBL-11687 Rev.
EEB-W-80-16
W-71

Presented at the Illuminating Engineering Society Annual Technical Conference, Dallas TX, August 24-28, 1980, and published in the Journal of the IES, July 1981, pp. 219-227.

CALCULATING INTERIOR DAYLIGHT ILLUMINATION WITH A PROGRAMMABLE HAND CALCULATOR

Harvey J. Bryan* and Robert D. Clear

Energy Efficient Buildings Program
Lawrence Berkeley Laboratory
University of California
Berkeley CA 94720

July 1981

*Department of Architecture
Massachusetts Institute of Technology
Cambridge MA 02139

This work was supported by the Assistant Secretary for Conservation and Renewable Energy, Office of Building Energy Research and Development, Building Systems Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

NOTE

This report was first reprinted September 1981, with a correction in Equation 5. This printing (April 1983) corrects errors in Tables 1 and 3 and, for user convenience, provides fits for the clear sky normalization and window factors equations.

A complete program listing for the TI-59 calculator, based on the procedure described herein, appeared in Lighting Design and Application, June 1981, pp. 28-36 and in Solar Age, August 1981, pp. 37-47. It also appears in Lawrence Berkeley Laboratory Report LBL-12248.

This work was conducted as part of the Lawrence Berkeley Laboratory's Windows and Daylighting Program with support provided by the U.S. Department of Energy. For further information regarding related activities, please contact:

Windows and Daylighting Program
Building 90, Room 3111
Lawrence Berkeley Laboratory
Berkeley CA 94720
(415) 486-5605

LBL-11687
EEB-W-80-16
W-71

ERRATA

QUICKLITE 1 Daylighting Program

The documentation for the TI-59 daylighting program, now called QUICKLITE 1, contains a table of clear sky normalizations ($N_{\delta c}$) on page 17 and a table of clear sky window factors ($f_.$) on page 20. Appropriate values from these tables are used as input to the clear sky calculations. Both tables are incorrect. The correct values are given here in Tables 1 and 2. Two sets of fits which may be easier to use than the tabulated values are given in Tables 3 and 4.

The normalization factor is described in the QUICKLITE 1 documentation as the ratio of the illumination on a horizontal plane, E_0, to the luminance at the zenith, L (0). The illumination at a point inside a room, E_p, is:

$$E_p = \int_{\phi_\ell}^{\phi_u} \int_{\theta_\ell}^{\theta_u} L(\theta,\phi)\ (T(\theta,\phi)\ \cos\theta)\ \sin\theta\ d\theta\ d\phi \qquad (1)$$

where $L(\theta,\phi)$ is the sky luminance at (θ,ϕ), $\cos(\theta)\ T(\theta,\phi)$ is the correction for the projected surface area and window transmission, and $\sin\theta\ d\theta\ d\phi$ is the solid angle element. For a clear sky, $L(\theta,\phi)$ factors as $(F_1(\theta,\phi)\ L(0))/F_2$ where F_2 is not dependent on (θ,ϕ).[1] QUICKLITE 1 takes advantage of this factorization by evaluating E_p for the clear sky as:

$$E_p = \frac{E_0}{N_{\delta c}} \int_{\phi_\ell}^{\phi_u} \int_{\theta_\ell}^{\theta_u} F_1(\theta,\phi)\ (T(\theta,\phi)\ \cos\theta)\ \sin\theta\ d\theta\ d\phi \qquad (2)$$

Solving for the normalization factor, $N_{\delta c}$ gives:

$$N_{\delta c} = (E_0\ F_2)/L(0). \qquad (3)$$

Thus, for a clear sky the definition given in Reference 1 is incorrect, because $N_{\delta c}$ must include the extra factor F_2.

In the QUICKLITE 1 documentation the tabulated $N_{\delta c}$ values are

approximately equal to $1/2(E_0/L(0))$. The added factor-of-2 error gave a normalization at a solar altitude of 40° (our worked example) that is unfortunately accurate to about 2% of the correct value.

The window factor is defined as the ratio of the illumination per unit surface area on a vertical surface ($\sin\theta \cos\phi$) to the illumination per unit surface area on a horizontal surface ($\cos\theta$):

$$f_\delta = \frac{\int_{-\pi/2}^{\pi/2} \left(\int_{0}^{\pi/2} L(\theta,\phi) \sin^2\theta \, d\theta \right) \cos\phi \, d\phi}{\int_{-\pi}^{\pi} \left(\int_{0}^{\pi/2} L(\theta,\phi) \sin\theta \cos\theta \, d\theta \right) d\phi} \qquad (4)$$

The window factor in the original documentation was calculated without changing from $\cos\theta$ to $\sin\theta\cos\phi$ in the numerator.

The effect of the above errors is to multiply the sky and reflected components by the wrong factors. We have computed new values for $N_{\delta C}$ and f_δ using Simpson's rule to estimate the double integrals. The step size was decreased until the values changed by less than .01%. The new values are presented in Tables 1 and 2. Tables 3 and 4 present simple fits to a more complete table of results. Note that the fits in Table 3 are more accurate than the basic numerical-integral, split-flux procedures used in QUICKLITE 1 and should therefore be adequate for almost all purposes.

A set of five runs at different solar altitudes was checked against equivalent runs using the program SUPERLITE.[2] Using the new values of $N_{\delta C}$ and f_δ, the maximum discrepancies appear at or opposite the window and range from 15-25%. The root-mean square (average) discrepancy for all points was 11%. Results averaged 6% higher with QUICKLITE than with SUPERLITE. Using the original values of $N_{\delta C}$ and f_δ, the results differed from those of SUPERLITE by as much as a factor of 6 when the solar altitude was 90°. Our original example was unfortunately accurate to 5% -- a triumph for Murphy's Law instead of a validation of the program and documentation as we had believed.

A precision of 10-20% seems consistent with the algorithms used in QUICKLITE 1, and we hope that this comparison indicates that there are no further major problems in the program. In the future we do expect to do more validation tests for both programs.

ACKNOWLEDGEMENTS

We wish to thank Dr. Tadahiko Nagata of Fukui University, Japan (presently on sabbatical leave at Lawrence Berkeley Laboratory) for calling our attention to the error in the sky normalization values.

This work was supported by the Assistant Secretary for Conservation and Renewable Energy, Office of Buildings and Community Systems, Buildings Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

REFERENCES

1. Bryan, Harvey J. and Clear, Robert D. 1981. Calculating interior daylight illumination with a programmable hand calculator. Journal of the Illuminating Engineering Society 10(4), July:219-227.
2. Modest, Michael F. 1981. Daylighting calculations for non-rectangular interior spaces with shading devices. LBL Report No. 12599. Berkeley, CA: Lawrence Berkeley Laboratory.

Table 1

Clear Sky Normalization Factors

Solar Altitude	$N_{\delta c}$
10°	2.129
20°	2.353
30°	2.547
40°	2.702
50°	2.820
60°	2.906
70°	2.964
80°	2.998
90°	3.009

Table 2

Clear Sky Window Factors (f_δ)

Solar Altitude	Window Azimuth from Sun				
	0°	45°	90°	135°	180°
10°	1.54	1.23	0.65	0.50	0.51
20°	1.41	1.13	0.60	0.45	0.44
30°	1.23	1.00	0.56	0.41	0.40
40°	1.05	0.87	0.52	0.38	0.37
50°	0.89	0.75	0.49	0.37	0.35
60°	0.74	0.65	0.47	0.37	0.35
70°	0.62	0.56	0.45	0.38	0.36
80°	0.52	0.49	0.44	0.40	0.39
90°	0.44	0.44	0.44	0.44	0.44

Table 3

First Fits to Clear Sky Normalizations ($N_{\delta C}$) and Window Factors (f_δ)

$$N_{\delta C}{}^{\dagger} = \sum_{i=0}^{2} A_i X^i$$

Table of A_i

i	0	1	2
A_i	2.7604	.5538	-.3139

Maximum error* = 0.6% at $\theta_\delta = 0°$.

Root-mean square (average) error* = 0.3%.

$$f_\delta{}^{\dagger} = \sum_{i=0}^{2} \sum_{j=0}^{2} B_{ij} X^i Y^j$$

Table of B_{ij}

	j		
i	0	1	2
0	.5124	.3176	.1632
1	-.1438	-.2777	-.2143
2	.0701	-.0654	.0220

Maximum error* = 12.9% at $\theta_\delta = 90°$, $\phi = 0°$.

Root-mean square (average) error* = 2.8%.

---

†X = $(\theta_\delta - 45)/45$ where θ_δ = solar altitude in degrees.
Y = $\cos(\phi)$ where ϕ = window azimuth from sun.

*The numerical evaluations of $N_{\delta C}$ and f_δ are presumed to be accurate to ~0.01%. The quoted errors are the errors of the fit relative to these numerical estimates.

Table 4

Second Fits to Clear Sky Normalizations ($N_{\delta c}$) and Window Factors (f_δ)

$$N_{\delta c}{}^{\dagger} = \sum_{i=0}^{4} A_i X^i$$

Table of A_i

i	0	1	2	3	4
A_i	2.766521	.547665	-.369832	.009237	.059229

Maximum error* = 0.3% at $\theta_\delta = 0°$.

Root-mean square (average) error* = 0.1%.

$$f_\delta{}^{\dagger} = \sum_{i=0}^{4} \sum_{j=0}^{4} B_{ij} X^i Y^j$$

Table of B_{ij}

	j				
i	0	1	2	3	4
0	.505551	.305110	.184461	0	-.029975
1	-.142919	-.357353	-.384803	0	.093932
2	.089048	.048448	.177271	0	-.076023
3	.010839	.119970	.158878	0	-.043539
4	-.022966	-.120651	-.141648	0	.057687

Maximum error* = 2.1% at $\theta_\delta = 90°$, $\phi = 0°$.

Root-mean square (average) error* = 0.6%.

$^{\dagger}X = (\theta_\delta - 45)/45$ where θ_δ = solar altitude in degrees.
$Y = \cos(\phi)$ where ϕ = window azimuth from sun.

*The numerical evaluations of $N_{\delta c}$ and f_δ are presumed to be accurate to ~0.01%. The quoted errors are the errors of the fit relative to these numerical estimates.

CALCULATING INTERIOR DAYLIGHT ILLUMINATION WITH A PROGRAMMABLE HAND CALCULATOR

Harvey J. Bryan and Robert D. Clear

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

ABSTRACT

A procedure is described for calculating interior daylight illumination using an inexpensive programmable hand calculator. The proposed procedure calculates illumination at any point within a room utilizing sky luminance distribution functions that are consistent with the CIE (Commission Internationale de l'Eclairage) Overcast and Clear Sky functions. This procedure separates the light reaching the point being considered into three components, these being (a) light directly from the sky, (b) light after being reflected from external, and (c) internal surfaces. Finally, two examples are presented in order to demonstrate the proposed procedure and indicate the speed with which the calculations may be performed.

KEY WORDS

Daylighting, natural illumination, lighting, energy conservation, energy use in buildings, programmable hand calculators.

INTRODUCTION

As design professionals focus on thermal load reduction in commercial buildings, lighting looms as the single largest energy consumer in these buildings. Daylighting is presently being proposed as one of the most promising energy conservation strategies for reducing these lighting loads.[1] Although substantial savings in both electrical energy and peak power demand exist, it is not obvious that projected savings can be achieved given the present state of daylighting design procedures. The major obstacle in the utilization of daylighting is the lack of accurate and simple procedures for daylighting analysis.

Present procedures for calculating the effects of daylight can be divided into two categories: simplified procedures (e.g., lumen method[2]) which often make assumptions which can result in questionable accuracy, and computer programs (e.g., Lumen II daylighting program[3]) which although highly accurate, require the preparation of detailed input data and access to time-sharing computer facilities. Therefore, it became necessary that a procedure be developed for calculating daylight which is highly accurate and yet not dependent on a computer environment. This objective can now be achieved with the aid of a programmable hand calculator.

Numerous procedures for calculating illumination from natural sources have been discussed in the literature.[4] However, none of these procedures have been introduced into a programmable hand calculator. The proposed procedure utilizes the daylight factor method, which is able to calculate daylight illumination at any point within a room. The procedure presented in this paper was developed specifically for the Texas Instruments TI-59 calculator, however with some minor modifications this procedure can be extended to other programmable hand calculators (e.g., HP-67).

THE DAYLIGHT FACTOR METHOD

The daylight factor method, which is recommended by the CIE (Commission Internationale de l'Eclairage), is used in Europe, particularly in Britain where the method was first developed. Today, the daylight factor is defined as the ratio between the daylight illumination at a point in the interior and the simultaneous exterior illumination available on a horizontal surface from an unobstructed sky (excluding direct sunlight) expressed as a percentage. The light reaching the point being considered is separated into three paths (see Figure 1): light directly from the sky (Sky Component or SC); light after reflection from external surfaces (Externally Reflected Component or ERC); and light after reflection from internal surfaces (Internally Reflected Component or IRC). The total for these three components gives the daylight factor, which is simply expressed as:

$$\text{Daylight Factor} = SC + ERC + IRC$$

THE SKY COMPONENT

The sky component is the ratio between the daylight illumination at a point in the interior which is received directly from the sky and the simultaneous exterior illumination available on a horizontal surface from an unobstructed sky.

Overcast Sky:

The sky component from an overcast sky is a function of the sky luminance, the window transmission, the solid angle ($\sin\theta d\theta d\phi$) and the cosine correction ($\cos\theta$) for the projection onto the horizontal plane. Here θ, ϕ are the standard polar angles; θ equals the angle from the zenith ($\theta = 90^{o}$-altitude) and ϕ equals the azimuthal angle. For convenience we set

ϕ=0 to be the direction of the window normal. The CIE Standard Sky luminance function[5] is given by the following formula:

$$L(\theta) = \frac{L(0)}{3} (1 + 2\cos\theta) \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where

$L(\theta)$ = Luminance of sky position at angle θ
$L(0)$ = Luminance of the zenith
θ = Angle from the zenith to sky position

The illumination through a window is then divided by the illumination on a horizontal plane which is the integral of equation (1) times the cosine correction over the hemisphere. This integral has been done analytically and is equal to $7\pi/9$ L(0).

The window transmission correction for angle of incidence loss must also be considered. Rivero's approximation for angle of incidence loss is used,[6] which is given by the following formula:

$$T(\Psi) = 1.018\ T(0)\ \cos^{\Psi}(1 + \sin^3\Psi) \quad \ldots\ldots\ldots\ldots (2)$$

where

$T(\Psi)$ = Transmission of glass at angle Ψ to the normal of the window
$T(0)$ = Transmission at normal incidence
Note: The Rivero approximation actually gives a slightly higher transmission value at normal incidence (Ψ=0) than the measured value, the reason for this is because it optimizes over all angles.

The sky component program does the integral over the product of equations (1), (2) and the cosine correction numerically. The logic of the program is made simpler by changing to a rectangular coordinate system Ω, ϕ (see Figure 2) so that the limits of the integral are independent and are given by:

$$\Omega = \tan^{-1}(\tan\theta\cos\phi)$$

which gives the following angular relationships;

$$\cos\theta = \cos\Omega \cos\phi / x^{\frac{1}{2}}$$

$$\sin\theta = \sin\Omega / x^{\frac{1}{2}}$$

$$\cos\Psi = \cos\phi \sin\Omega / x^{\frac{1}{2}}$$

$$\sin\Psi = \left(\sin^2\Omega + \cos^2\varphi\,(\cos^2\Omega - \sin^2\Omega)\right)^{1/2}/x^{1/2}$$

$$d\theta = \cos\varphi\, d\Omega/x; \quad d\varphi = d\varphi$$

where $x = \cos^2\Omega\cos^2\varphi + \sin^2\Omega$ and $-\frac{\pi}{2} < \varphi < \frac{\pi}{2}$

The integral is derived from equations (1), (2) and the above angular relationships. The integral is given as follows:

$$SC = \frac{3.054T(0)}{7\pi}\int_{\Omega_L}^{\Omega_U}\int_{\varphi_L}^{\varphi_M}\left(\frac{\sin^2\Omega\cos\Omega\cos^3\varphi}{x^{5/2}}\right)\left(1+\frac{2\cos\varphi\cos\Omega}{x^{1/2}}\right) \times \left(1+\left(\frac{\sin^2\Omega+\cos^2\varphi\,(\cos^2\Omega-\sin^2\Omega)}{x}\right)^{3/2}\right) d\varphi\, d\Omega \quad \ldots\ldots(3)$$

The limits of the integral are given by the angular coordinates of the four corners of the window. The integral over φ is performed using the calculator's built in Simpson's rule integration routine (TI-59's Master Library Program 09). The integral over Ω is performed by writing a separate Simpson's rule routine.

Clear Sky:

The clear sky component calculation for the clear sky has the same form as the overcast sky calculation except for the sky luminance function and its associated inputs. The clear sky luminance function that will be used is consistent with the function presently being considered by the CIE Technical Committee TC-4.2,[7] which is given by the following formula:

$$\frac{L(\theta)}{L(0)} = \frac{F_1}{F_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

where

$L(\theta)$ = Luminance of sky position at angle θ
$L(0)$ = Luminance of the zenith
$F_1 = (1-e^{-0.32\sec\theta})(0.91+10e^{-3\delta}+0.45\cos^2\delta)$
$F_2 = 0.274(0.91+10e^{-3\theta_s}+0.45\cos^2\theta_s)$
θ = Angle from the zenith to sky position
θ_s = Angle from the zenith to the sun
δ = Angle of the sun from $L(\theta)=\cos^{-1}(\sin\theta\,\sin\theta_s\,\cos(\varphi+\varphi_w-\varphi_s)+\cos\theta\,\cos\theta_s)$

In addition the sky component calculation for the clear sky requires added inputs: θ_s, the angular zenith distance of the sun; ϕ_s, the angular azimuthal distance of the sun from south; ϕ_w, angular azimuthal distance of the window normal from south; and Nsc, normalization factor which is the illumination on a horizontal plane from the hemisphere times the factor F_2 (see Equation 4) divided by the zenith luminance, L(0). This normalization factor has been integrated numerically and is documented in Table 1. Fits for the clear sky normalization factors are presented, for user convenience, in Tables A and B.

As in the overcast sky program the integration is done over a rectangular coordinate system by an integral derived from equations (2), (4) and the previously presented angular relationships. The integral is given as follows:

$$SC = \frac{1.018T(0)}{Nsc} \int_{\Omega_l}^{\Omega_u} \int_{\phi_l}^{\phi_u} \left(\frac{\sin^2\Omega \cos\Omega \cos^3\phi}{x^{5/2}} \right) \times \left(1 + \left(\frac{\sin^2\Omega + \cos^2\phi\,(\cos^2\Omega - \sin^2\Omega)}{x} \right)^{3/2} \right) \left(1 - e^{-\frac{0.32\,x^{1/2}}{\cos\Omega\cos\phi}}\right) \times \left(0.91 + 10e^{-3\delta} + 0.45\cos^2\delta\right) d\phi\, d\Omega \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where $\delta = \cos^{-1}\left(\frac{1}{x^{1/2}} \left(\sin\Omega \sin\theta_s \cos(\phi + \phi_w - \phi_s) + \cos\Omega \cos\theta_s \cos\phi \right) \right)$

The limits of the integral are again given by the angular coordinates of the four corners of the window. The integral over ϕ is performed using the calculator's built in Simpson's rule integration routine (TI-59's Master Library Program 09). The integral over Ω is performed by writing a separate Simpson's rule routine.

THE EXTERNALLY REFLECTED COMPONENT

The externally reflected component is the ratio between the daylight illumination at a point in the interior which is received directly from external surfaces and the simultaneous exterior illumination available on

a horizontal surface from an unobstructed sky.

Overcast Sky:

The externally reflected component from an overcast sky can be determined by considering the external obstruction that is visible from the reference point as an effective sky component (SC_e) whose luminance is some fraction of the sky that is obscured. The effective sky component for any obstruction is calculated by the Sky Component Program for the overcast sky, this value is then converted into the externally reflected component by correcting for the reduced luminance of the obstruction compared with the luminance of the sky. In cases where the obstruction is below 20^o, its luminance is usually assumed to be one fifth that portion of the sky that is obscured. For such cases, the effective sky component of the obstruction is divided by 5, so that the ERC is:

$$ERC = \frac{SC_e}{5}$$

In cases where the obstruction is above 20^o, its luminance is usually assumed to be one tenth that portion of the sky that is obscured. For such cases, the effective sky component of the obstruction is divided by 10, so that the ERC is:

$$ERC = \frac{SC_e}{10}$$

Clear Sky:

The externally reflected component from a clear sky has not yet been introduced into the programmable hand calculator procedure. However, horizontal shading devices can be modeled by assuming an effective window and neglecting the luminance of the under side of the device, which is usually

small. Work is in progress to extend the programmable hand calculator procedure to include the externally reflected component for more complex obstructions. The present procedure may be used with no loss in accuracy when there are no external obstructions visible from the reference point.

THE INTERNALLY REFLECTED COMPONENT

The internally reflected component is the ratio between the daylight illumination at a point in the interior which is received after being interreflected off interior surfaces and the simultaneous exterior illumination available on a horizontal surface from an unobstructed sky. Presently there are two approaches for calculating the internally reflected component, the split flux method and the more accurate finite difference method. The approach that will be used here is the split flux method, the simpler of the two.

The split flux method divides the light entering the room into two parts (see Figure 3), light received directly from the sky and that received directly from the ground. The light from the sky on entering the room is considered to be modified by the average reflectance of the floor and those parts of the walls below the mid-height of the window. The light from the ground is considered to be modified by the average reflectance of the ceiling and those parts of the walls above the mid-height of the window.

The formula for the average IRC is given as:[8]

$$IRC = \frac{T \times W}{A(1-R)} (f_s \cdot R_{fw} + f_g \cdot R_{cw}) \times 100\% \quad \ldots\ldots\ldots\ldots (6)$$

where

T = Transmittance of glass
W = Area of window
A = Total area of ceiling, floor and walls including area of window
R = Average reflectance of the ceiling, floor and all walls, including window
R_{fw} = Average reflectance of the floor and those parts of the walls,

below the plane of the mid-height of the window (excluding the window-wall)

R_{cw} = Average reflectance of the ceiling and those parts of the walls, above the plane of the mid-height of the window (excluding the window-wall)

f_s = Window factor due to the light incident on the window from sky

f_g = Window factor due to the light incident on the window from ground

Overcast Sky:

The internally reflected component from an overcast sky is determined by equation (6). The various window (transmittance and area) and room (surface areas and reflectances for walls, ceiling and floor) conditions are determined from the design, while the window factors are determined as follows; f_s from Table 2 for various angles of obstructions, and f_g by multiplying the reflectances of ground surface (R_g) by the ground configuration factor (G_{cf}) which is always 0.5 for a horizontal ground plane. The various values are then entered into the IRC program which calculates the average internally reflected component for the entire room.

Clear Sky:

The internally reflected component from a clear sky is also determined by equation (6). However, because direct solar illumination is contributing illumination to the ground, f_s and f_g are calculated differently. The window factors for f_s have been precalculated and are documented in Table 3 for various window orientations and solar altitudes, while f_g is determined as follows:

$$f_g = \frac{(E_{sun} + E_{sky}) \times R_g \times G_{cf}}{E_{sky}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

where

E_{sun} = Illumination from the sun (from Figure 37A in the Recommended Practice of Daylighting)

E_{sky} = Illumination from the sky (from Figure 36B, C, or D in the Recommended Practice of Daylighting)

R_g = Reflectance of ground plane
G_{cf} = Ground configuration factor (for a horizontal surface this value is always 0.5)

The various values are then entered into the IRC program which calculates the average internally reflected component for the entire room.

Fits for the clear sky window factors equation are presented in Tables A and B, for user convenience.

THE TOTAL DAYLIGHT FACTOR

The total daylight factor is the sum of the sky component (SC), the externally reflected component (ERC) if present and the internally reflected component (IRC) programs. However, in order that the daylight factor be useful it needs to be converted into daylight illumination. This is accomplished by determining the exterior sky illumination incident on a horizontal surface, from Figures 36B, C, or D in the Recommended Practice of Daylighting[2] or some other daylight availability reference; this value is then multiplied by the daylight factor in order to get the interior daylight illumination.

VALIDATION OF PROPOSED PROCEDURE

Both the overcast and clear sky programmable hand calculator procedures have been validated against the Lumen II computer program. The Lumen II program was selected because it is well documented, has undergone extensive testing, and its daylighting features have been compared favorably to a series of physical model measurements.[3]

Figure 4 illustrates the 20'x20'x10' room with 6'x16' window 2.5' high that was used in both the overcast and clear sky examples. Within this room a 2.5'x2.5' grid (49 reference points) 2.5' high was selected for analyses. External and internal conditions were then established as criteria for these examples.

The external conditions for these examples assumed the following para-

meters.

Date: June 21
Time: 8:20
Area Condition: Rural
Climate Condition: Temperate
Sky Condition: Overcast and Clear
Solar Altitude: 40°
Solar Azimuth: 90°
Window Orientation: 90° west of sun
Illumination from sky on ground: 1268 footcandles from overcast sky, 1374 footcandles from clear sky (Lumen II values)
Illumination from sun on ground: 5192 footcandles for clear sky only (Lumen II value)
Ground Reflectance: 20%
Obstruction: None

The internal conditions for these examples assumed the following parameters.

Room Length: 20 feet
Room Width: 20 feet
Room Height: 10 feet
Sill Height: 2.5 feet
Ceiling Reflectance: 70%
Wall Reflectance: 60%
Floor Reflectance: 40%
Glazing Type: Clear, 85% Transmittance

Overcast Sky:

The sky component program took approximately 5 minutes for data entry and another 60 minutes (1.2 minutes/reference point) to calculate the 49 reference points (It should be pointed out that for a room of this size, 4 to 9 reference points would be sufficient for daylighting analysis). However the designer does not need to be present for the calculation phase if a PC-100 printer is used. The results of the sky component program are presented in Table 4.

The externally reflected component was omitted because no obstructions are present.

The internally reflected component program took approximately 4 minutes for data entry and another minute to calculate the average IRC.

The daylight factor was then determined by adding the calculated internally reflected component of 2.6% to the 49 sky component values. Finally, the daylight factors are multiplied by the horizontal illumination available from an unobstructed overcast sky (1268 footcandles) in order to determine the daylight illumination incident on the reference points. The daylight illumination for the 49 reference points are presented in Table 5.

A Lumen II daylighting analysis was then performed utilizing the same parameters as was assumed in the programmable hand calculator procedure. The daylighting illumination for the 49 reference points are presented in Table 6.

The percent difference between the Programmable Hand Calculator Procedure and Lumen II were calculated and are presented in Table 7. From Table 7 it can be observed that except for the 7 reference points in the row along the window (which can be explained by the fact that Lumen II does not correct for angle of incidence losses), the Programmable Hand Calculator Procedure results are within $\pm$ 5% of Lumen II results. The difference between the averages is 3.4 footcandles, the ratio of the averages is .97, and the standard deviations for the Programmable Hand Calculator Procedure and Lumen II are 74.83 and 81.79 respectively. These results suggest a satisfactory correlation between the proposed procedure and Lumen II for this example.

Clear Sky:

The sky component program took approximately 5 minutes for data entry and another 150 minutes (3 minutes/reference point) to calculate the 49 reference points (Again it should be pointed out that for a room of this size, 4 to 9 reference points would be sufficient for daylighting analysis).

Again the designer does not need to be present for the calculation phase if a PC-100 printer is available. The results of the sky component program are presented in Table 8.

The externally reflected component was omitted because no obstructions are present.

The internally reflected component program took approximately 4 minutes for data entry and another minute to calculate the average IRC.

The daylight factor was determined by adding the calculated internally reflected component of 6.4% to the 49 sky component values presented in Table 8. Finally, the daylight factors are multiplied by the horizontal illumination available from an unobstructed clear sky (1374 footcandles) in order to determine the daylight illumination incident on the reference points. The daylight illumination for the 49 reference points are presented in Table 9.

A Lumen II daylighting analysis was then performed utilizing the same parameters as was assumed in the programmable hand calculator procedure. The daylighting illumination for the 49 reference points are presented in Table 10.

The percent difference between the Programmable Hand Calculator Procedure and Lumen II were calculated and are presented in Table 11. From Table 11 it can be observed that except for 8 reference points in the rear corner of the room and again for the reference points along the window the Programmable Hand Calculator Procedure results are within $\pm$ 5% of Lumen II results. The difference between the averages is 5.7 footcandles, the ratio of the averages is .97, and the standard deviations for the Programmable Hand Calculator Procedure and Lumen II are 72.10 and 78.30 respectively. Here again the results suggest a satisfactory correlation between the proposed procedure and Lumen II for this example.

An accurate, simple and relatively fast procedure for calculating daylight has been presented which links itself with internationally recommended practices. It is hoped that the application of this procedure will encourage the use of daylighting, as well as placing daylighting in a proper relationship to other design considerations. A lengthier version of this paper which includes a detailed program description, program listings, user instructions and several worked examples is available from the authors at: Windows and Daylighting Program, Lawrence Berkeley Laboratory, Building 90, Room 3111, Berkeley, California, 94720.

REFERENCES

1. U.S. Department of Energy, Office of Conservation and Solar Energy, "Energy Performance Standards for New Buildings; Proposed Rule, Part II," Federal Register, Vol. 44, No. 230, November 28, 1979, p. 68135.
2. IES Daylighting Committee, "Recommended Practice of Daylighting," LIGHTING DESIGN & APPLICATION, Vol. 9, No. 2, February 1979, p. 45-58.
3. DiLaura, D.L., and Hauser, G.A., "On Calculating the Effects of Daylighting in Interior Spaces," JOURNAL OF THE ILLUMINATING ENGINEERING SOCIETY, Vol. 8, No. 1, October 1978, p. 2.
4. CIE Technical Committee E-3.2, "Daylight: International Recommendations for the Calculation of Natural Daylight," CIE PUBLICATION No. 16, Commission Internationale de l'Eclairage, Paris, 1970, p. 48-79.
5. CIE Technical Committee E-3.2, "Natural Daylight: Official Recommendations," Commission Internationale de l'Eclairage, 13th Session, Paris, Vol. 2, 1955, p. II.
6. Rivero, R. "Illuminacion Natural. Calculo del Factor de Dia Directo para Ventanas sin Vidrios y con Vidrios y para Cielos Uniformes y No Uniformes," Instituto de la Construccion de Edificios, Dpto. de Acondicionamiento, Facultad de Arquitectura, Montevideo, 1958.
7. CIE Technical Committee 4.2, "Standardization of Luminance Distribution on Clear Skies," CIE PUBLICATION No. 22, Commission Internationale de l' Eclairage, Paris, 1973, p. 7.
8. Hopkinson, R.G., Longmore, J., and Petherbridge, P., "An Empirical Formula for the Computation of the Indirect Component of the Daylight Factor," TRANSACTIONS OF THE ILLUMINATING ENGINEERING SOCIETY, London, Vol. 19, 1954, p. 201.

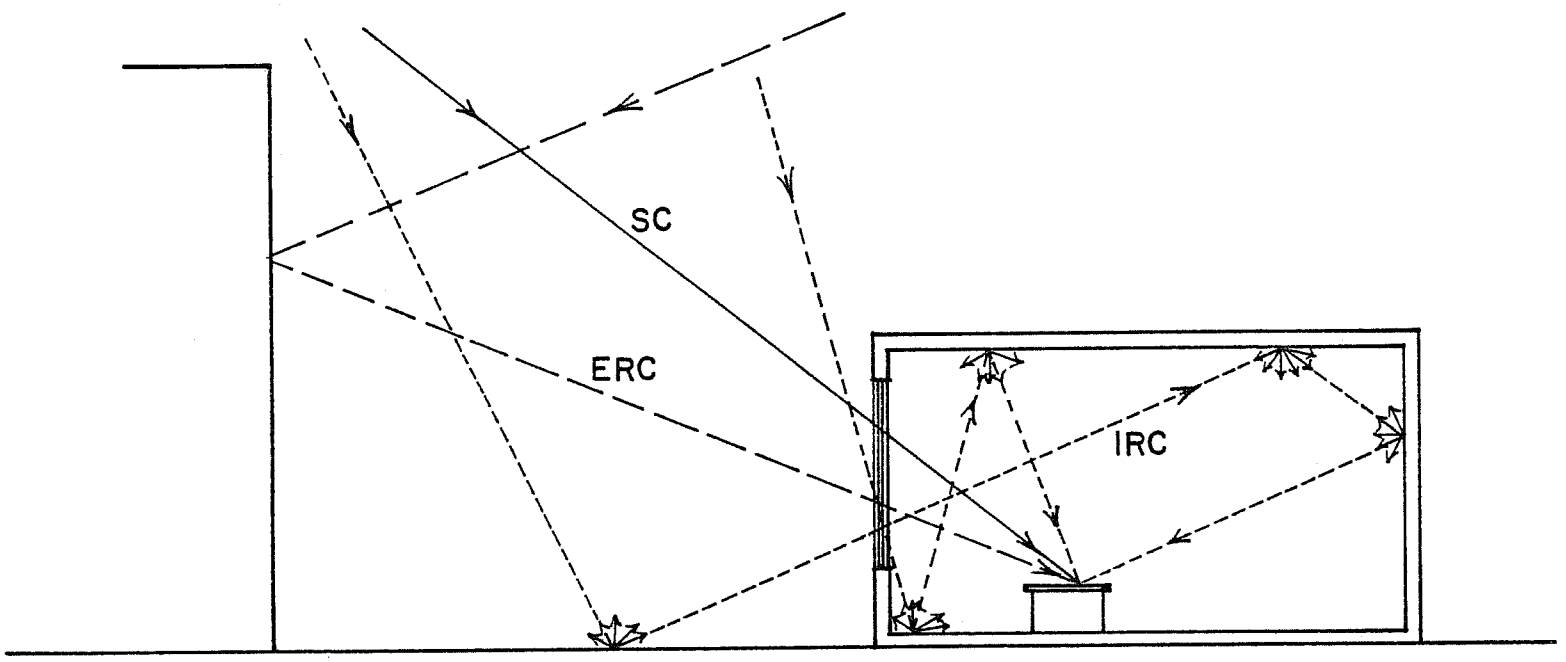

Figure 1. Components of the daylight factor

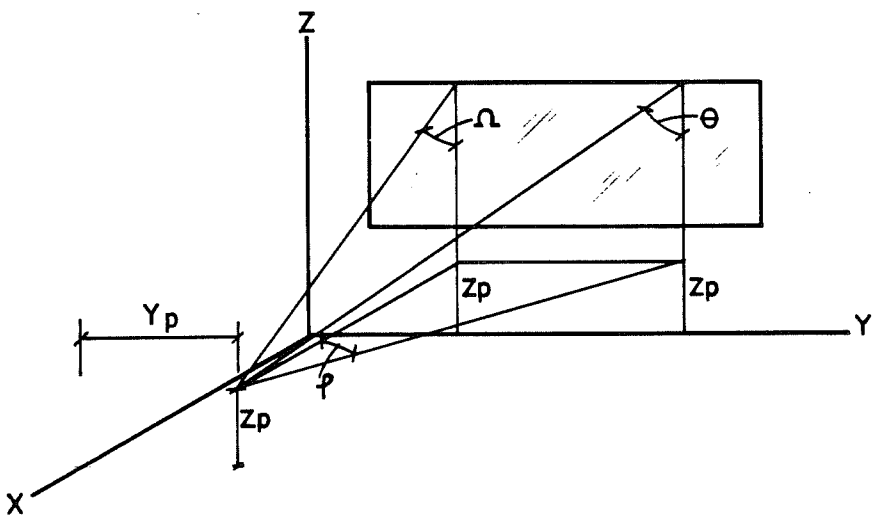

Figure 2. Angular coordinate system of window

Table 1

Clear Sky Normalization Factors

Solar Altitude	$N_{\delta c}$
10°	2.129
20°	2.353
30°	2.547
40°	2.702
50°	2.820
60°	2.906
70°	2.964
80°	2.998
90°	3.009

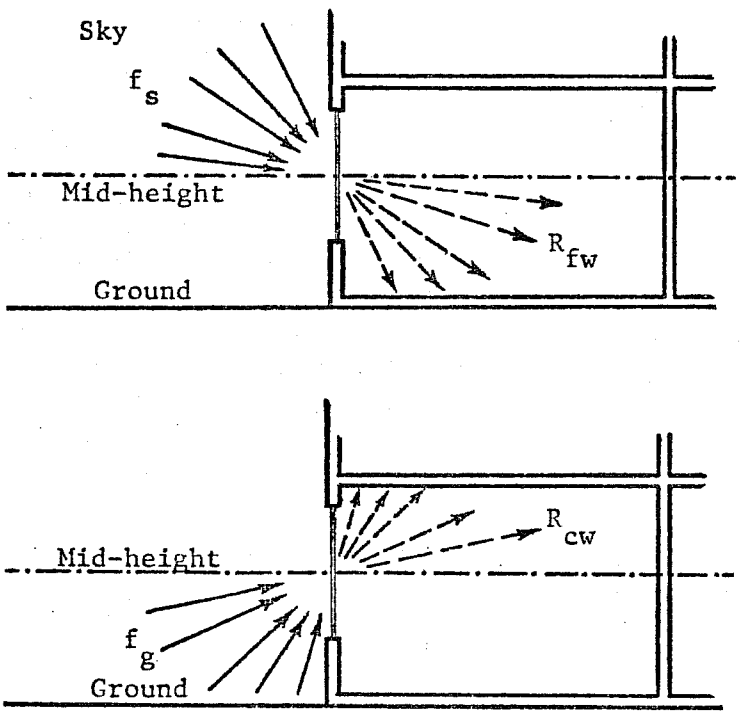

Figure 3. Conceptual illustration of the split flux method

Table 2.

Angle of obstruction measured from center of window	f_s
no obstruction	.39
10^o	.35
20^o	.31
30^o	.25
40^o	.20
50^o	.14
60^o	.10
70^o	.07
80^o	.05

Table 3

Clear Sky Window Factors (f_δ)

Solar Altitude	Window Azimuth from Sun				
	0°	45°	90°	135°	180°
10°	1.54	1.23	0.65	0.50	0.51
20°	1.41	1.13	0.60	0.45	0.44
30°	1.23	1.00	0.56	0.41	0.40
40°	1.05	0.87	0.52	0.38	0.37
50°	0.89	0.75	0.49	0.37	0.35
60°	0.74	0.65	0.47	0.37	0.35
70°	0.62	0.56	0.45	0.38	0.36
80°	0.52	0.49	0.44	0.40	0.39
90°	0.44	0.44	0.44	0.44	0.44

Table A

First Fits to Clear Sky Normalizations ($N_{\delta C}$) and Window Factors (f_δ)

$$N_{\delta C}{}^{\dagger} = \sum_{i=0}^{2} A_i X^i$$

Table of A_i

i	0	1	2
A_i	2.7604	.5538	-.3139

Maximum error* = 0.6% at $\theta_\delta = 0°$.

Root-mean square (average) error* = 0.3%.

$$f_\delta{}^{\dagger} = \sum_{i=0}^{2} \sum_{j=0}^{2} B_{ij} X^i Y^j$$

Table of B_{ij}

	j		
i	0	1	2
0	.5124	.3176	.1632
1	-.1438	-.2777	-.2143
2	.0701	-.0654	.0220

Maximum error* = 12.9% at $\theta_\delta = 90°$, $\phi = 0°$.

Root-mean square (average) error* = 2.8%.

---

†X = $(\theta_\delta - 45)/45$ where θ_δ = solar altitude in degrees.
Y = $\cos(\phi)$ where ϕ = window azimuth from sun.

*The numerical evaluations of $N_{\delta C}$ and f_δ are presumed to be accurate to ~0.01%. The quoted errors are the errors of the fit relative to these numerical estimates.

Table B

Second Fits to Clear Sky Normalizations ($N_{\delta C}$) and Window Factors (f_δ)

$$N_{\delta C}{}^{\dagger} = \sum_{i=0}^{4} A_i X^i$$

Table of A_i

i	0	1	2	3	4
A_i	2.766521	.547665	-.369832	.009237	.059229

Maximum error* = 0.3% at $\theta_\delta = 0°$.

Root-mean square (average) error* = 0.1%.

$$f_\delta{}^{\dagger} = \sum_{i=0}^{4} \sum_{j=0}^{4} B_{ij} X^i Y^j$$

Table of B_{ij}

	j				
i	0	1	2	3	4
0	.505551	.305110	.184461	0	-.029975
1	-.142919	-.357353	-.384803	0	.093932
2	.089048	.048448	.177271	0	-.076023
3	.010839	.119970	.158878	0	-.043539
4	-.022966	-.120651	-.141648	0	.057687

Maximum error* = 2.1% at $\theta_\delta = 90°$, $\phi = 0°$.

Root-mean square (average) error* = 0.6%.

$^{\dagger}X = (\theta_\delta - 45)/45$ where θ_δ = solar altitude in degrees.
$Y = \cos(\phi)$ where ϕ = window azimuth from sun.

*The numerical evaluations of $N_{\delta C}$ and f_δ are presumed to be accurate to ~0.01%. The quoted errors are the errors of the fit relative to these numerical estimates.

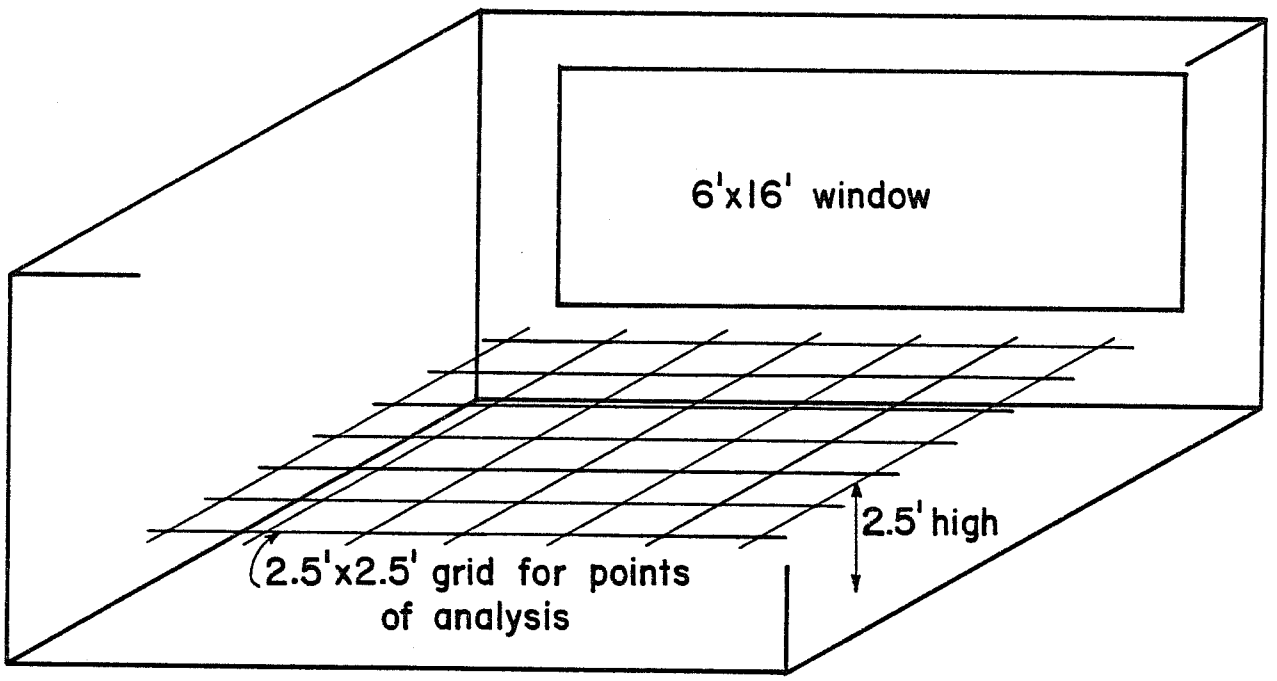

Figure 4. Typical room arrangement

Table 4.

****
PROGRAMMABLE HAND CALCULATOR PROCEDURE
****

SKY COMPONENT FOR OVERCAST SKY

WORKING PLANE HEIGHT: 2.50

ABS. Y COOR.	ABSOLUTE X-COORDINATE(S) 2.5	5.0	7.5	10.0	12.5	15.0	17.5
17.5 *	.6	.6	.7	.7	.7	.6	.6
15.0 *	.8	.9	1.0	1.1	1.0	.9	.8
12.5 *	1.2	1.5	1.7	1.7	1.7	1.5	1.2
10.0 *	2.0	2.5	2.9	3.0	2.9	2.5	2.0
7.5 *	3.5	4.7	5.4	5.6	5.4	4.7	3.5
5.0 *	6.5	9.3	10.6	10.9	10.6	9.3	6.5
2.5 *	12.3	18.5	20.0	20.2	20.0	18.5	12.3

Table 5.

****
PROGRAMMABLE HAND CALCULATOR PROCEDURE
****

ILLUMINATION FOR OVERCAST SKY

WORKING PLANE HEIGHT: 2.50

AVERAGE: 99.6 MINIMUM: 40.6 MAXIMUM: 289.1

STANDARD DEVIATION: 74.83

ABS. Y COOR.	ABSOLUTE X-COORDINATE(S) 2.5	5.0	7.5	10.0	12.5	15.0	17.5
17.5 *	40.6	40.6	41.8	41.8	41.8	40.6	40.6
15.0 *	43.1	44.4	45.6	46.9	45.6	44.4	43.1
12.5 *	48.2	52.0	54.5	54.5	54.5	52.0	48.2
10.0 *	58.3	64.7	69.7	71.0	69.7	64.7	58.3
7.5 *	77.3	92.6	101.4	104.0	101.4	92.6	77.3
5.0 *	115.4	150.9	167.4	171.2	167.4	150.9	115.4
2.5 *	188.9	267.5	286.6	289.1	286.6	267.5	188.9

Table 6.

****
LUMEN II DAYLIGHTING ANALYSIS
****

ILLUMINATION FOR OVERCAST SKY

WORKING PLANE HEIGHT: 2.50

AVERAGE: 103.0 MINIMUM: 42.0 MAXIMUM: 317.9

STANDARD DEVIATION: 81.79

ABS. Y COOR.	ABSOLUTE X-COORDINATE(S) 2.5	5.0	7.5	10.0	12.5	15.0	17.5
17.5	42.1	42.4	42.0	42.9	42.7	42.4	42.4
15.0	44.8	46.3	46.1	47.2	46.9	46.5	44.8
12.5	49.6	52.3	53.3	54.7	54.3	52.3	49.6
10.0	58.9	65.0	67.9	70.5	68.8	65.0	58.9
7.5	76.9	90.7	98.1	101.5	98.9	90.7	76.9
5.0	116.4	150.4	165.9	171.1	166.7	150.4	116.4
2.5	205.2	290.2	312.6	317.9	313.3	290.2	205.1

Table 7.

****
PERCENT DIFFERENCE BETWEEN PROPOSED PROCEDURE AND LUMEN II
****

FOR OVERCAST SKY

$$\text{PERCENT DIFFERENCE} = \frac{\text{PROPOSED PROCEDURE - LUMEN II}}{\text{LUMEN II}} \times 100\%$$

ABS. Y COOR.	ABSOLUTE X-COORDINATE(S) 2.5	5.0	7.5	10.0	12.5	15.0	17.5
17.5 *	-3.6	-4.2	-0.5	-2.6	-2.1	-4.2	-4.2
15.0 *	-3.8	-4.1	-1.1	-0.6	-2.8	-4.5	-3.8
12.5 *	-2.8	-0.6	+2.3	-0.4	+0.4	-0.6	-2.8
10.0 *	-1.0	-0.5	+2.7	+0.7	+1.3	-0.5	-1.0
7.5 *	+0.5	+2.1	+3.4	+2.5	+2.5	+1.9	+0.5
5.0 *	-0.9	+0.3	+0.9	+0.1	+0.4	+0.3	-0.9
2.5 *	-7.9	-7.8	-8.3	-9.1	-8.5	-7.8	-7.9

Table 8.

****
PROGRAMMABLE HAND CALCULATOR PROCEDURE
****

SKY COMPONENT FOR CLEAR SKY

WORKING PLANE HEIGHT: 2.50

ABS. Y COOR.	ABSOLUTE X-COORDINATE(S) 2.5	5.0	7.5	10.0	12.5	15.0	17.5
17.5 *	1.1	1.1	1.2	1.1	1.1	1.0	0.9
15.0 *	1.5	1.6	1.7	1.6	1.5	1.3	1.1
12.5 *	2.2	2.4	2.5	2.4	2.2	1.9	1.6
10.0 *	3.3	3.7	3.9	3.8	3.4	2.9	2.2
7.5 *	5.2	6.1	6.4	6.3	5.7	4.6	3.3
5.0 *	8.7	10.5	11.2	11.0	10.0	7.9	5.1
2.5 *	14.8	18.1	18.6	18.5	17.6	14.8	8.0

Table 9.

****
PROGRAMMABLE HAND CALCULATOR PROCEDURE
****

ILLUMINATION FOR CLEAR SKY

WORKING PLANE HEIGHT: 2.50

AVERAGE: 163.3 MINIMUM: 100.3 MAXIMUM: 343.5

STANDARD DEVIATION: 72.10

ABS. Y COOR.	ABSOLUTE X-COORDINATE(S) 2.5	5.0	7.5	10.0	12.5	15.0	17.5
17.5	103.1	103.1	104.4	103.1	103.1	101.7	100.3
15.0	108.5	109.9	111.3	109.9	108.5	105.8	103.1
12.5	118.2	120.9	122.3	120.9	118.2	114.0	109.9
10.0	133.3	138.8	141.5	140.1	134.7	127.8	118.2
7.5	159.4	171.8	175.9	174.5	166.3	151.1	133.3
5.0	207.5	232.2	241.8	239.1	225.3	196.5	158.0
2.5	291.3	336.6	343.5	342.1	329.8	291.3	197.9

Table 10.

****
LUMEN II DAYLIGHTING ANALYSIS
****

ILLUMINATION FOR CLEAR SKY

WORKING PLANE HEIGHT: 2.50

AVERAGE: 169.0 MINIMUM: 105.3 MAXIMUM: 379.3

STANDARD DEVIATION: 78.30

ABS. Y COOR.	ABSOLUTE X-COORDINATE(S) 2.5	5.0	7.5	10.0	12.5	15.0	17.5
17.5	111.3	111.1	107.8	108.8	107.6	105.8	105.3
15.0	118.7	117.1	114.4	115.4	113.5	111.6	107.8
12.5	127.2	127.7	123.0	124.6	122.0	117.3	112.6
10.0	141.6	144.5	141.6	141.2	136.6	128.9	120.7
7.5	166.1	173.9	172.9	171.9	161.8	148.3	132.3
5.0	215.9	235.3	239.2	236.6	220.0	189.7	154.3
2.5	322.2	371.2	379.3	376.0	353.2	297.2	196.6

Table 11.

****
PERCENT DIFFERENCE BETWEEN PROPOSED PROCEDURE AND LUMEN II
****

FOR CLEAR SKY

$$\text{PERCENT DIFFERENCE} = \frac{\text{PROPOSED PROCEDURE - LUMEN II}}{\text{LUMEN II}} \times 100\%$$

ABS. Y COOR.	ABSOLUTE X-COORDINATE(S) 2.5	5.0	7.5	10.0	12.5	15.0	17.5
17.5 *	-7.4	-7.2	-3.2	-5.2	-4.2	-3.9	-4.7
15.0 *	-8.6	-6.1	-2.7	-4.8	-4.4	-5.2	-4.4
12.5 *	-7.1	-5.3	-0.6	-3.0	-3.1	-2.8	-2.4
10.0 *	-5.9	-3.9	-0.1	-0.8	-1.4	-0.9	-2.1
7.5 *	-4.0	-1.2	+1.7	+1.5	+2.8	+1.9	+0.8
5.0 *	-3.9	-1.3	+1.1	+1.1	+2.4	+3.6	+2.4
2.5 *	-9.6	-9.3	-9.4	-9.0	-6.6	-2.0	+0.7